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SOLVING PORTFOLIO SELECTION PROBLEM USING PARTICLE SWARM OPTIMIZATION WITH CARDINALITY AND BOUNDING CONSTRAINTS

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ABSTRACT

The portfolio selection of assets for an investment by investors has remain a challenge in building appropriate portfolio of assets when investing hard earned money into different assets in order to maximize returns and minimize associated risk. Different models have been used to resolve the portfolio selection problem but with some limitations due to the complexity and instantaneity of the portfolio optimization model, however, particle swarm optimization (PSO) algorithm is a good alternative to meet the challenge.
This study applied cardinality and bounding constraints to portfolio selection model using a meta-heuristic technique of particle swarm optimization. The implementation of the developed model was done with python programming language. The results of this study were compared with that of the genetic algorithms technique as found in extant literature. The results obtained with the model developed shows that particle swarm optimization approach gives a better result than genetic algorithm in solving portfolio selection problem.

Keywords: *Portfolio, Genetic Algorithm, Particle Swarm Optimization, Cardinality and Bounding Constraints.*

1 INTRODUCTION

Portfolio selection problem is an interesting area in Economics and Artificial Intelligence research domain. The goal is to find the best approach of investing a particular amount of money into different assets in order to maximize returns and minimize risk. A portfolio consists of assets and investment capital. Portfolio selection has been a major challenge for both individual investors' and finance management companies. [1]
To solve this challenge, Markowitz introduced a model called the mean-variance model [2]. This model assumes that the total returns of a portfolio can be determined using the mean return of the assets and the risk between these assets. The sets of portfolios of assets that yield minimum risk for a given level of return form what is known as the efficient frontier [3] Markowitz model has gain a widespread use in the field of finance and has remained a general reference model in portfolio selection problem. [1].
However, as the size of the assets in portfolio increases, the standard Markowitz model becomes inefficient to optimize the returns because this model did not consider the cardinality and bounding constraint. Cardinality constraint has to do with the number of assets in the portfolio and ensures that the number of assets is not exceeded, while the bounding constraint takes care of or set the limit for the amount of money to be invested in each asset. These constraints are very necessary in the market scenario hence the Markowitz model has to be generalized in other to take care of these constraints. [3].
The rest of the paper is organized as follows. Section 2 presents related work. Section 3 is the research methodology while section 4 contained results and discussion obtained in this work and the paper is concluded in section 5.

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2 RELATED WORK

In literature, different models of particle swarm optimization (PSO) are identified and used in portfolio selection problem. To solve for the limitation of Markowitz mean-variance model, some other models were also used. These are: Constrained optimization (CO), Quadratic programming (QP), linear programming (LP), Second-order cone programming (SOCP) was developed and used [4]. These approaches are based on linear assumption; hence they are good for quadratic objective functions with a single objective. Meanwhile, these methods also have some constrains in portfolio optimization because they cannot solve non quadratic objective function in the sense that there exists more than one objective; maximization of returns and minimization of risk at the same time [5]

[6], in their publication titled constraint handling methods for portfolio optimization using particle swarm optimization, identified two portfolio constraint handling methods such as portfolio repair method and preserving feasibility method. Whose aim is to investigate which constraint handling techniques are better suited to the problem solved by applying particle swarm optimization.

[1] introduced a new constraint known as the expert opinion for portfolio selection that can address the real market scenarios such as change in the size of the portfolio, transaction cost etc. Also, they did a comparison between PSO and genetic algorithm, they obtained a result that shows that PSO performed better. [7] describe particle swarm optimization (PSO), as a co-operative population-based meta-heuristic algorithm for solving the Cardinality Constraints Markowitz Portfolio Optimization problem (CCMPO). To resolve the CCMPO challenge, the improved PSO is said to increase exploration in the original search that can improve convergence speed in the final search space. [8] described the modern portfolio theory (MPT) with the main objective of presenting how it enables investors to organise, evaluate, and regulate the rate of returns as well as the risk involved in portfolio optimisation. They also examined application of the theory to real time investment decisions relative to assumptions of the MPT.

[9] employs the Markowitz mean–variance model for portfolio selection problem to find a feasible portfolio with a minimum risk through the application of heuristic algorithm. He used two algorithm PSO and GA in his research. [10] describe an improved particle swarm optimization (IPSO) model for actual portfolio selection problem, which comprises of the total costs of the portfolio and a measure some constraints. The proposed method is an effective way of resolving the limitations in portfolio selection problem. IPSO is giving a better result when compared to standard PSO method. Since it can find better global optimum. Some other techniques has also been used, such as Fuzzy approach, Artificial Neural Network (ANN), Tabu Search (TS), Goal Programming (GP), Simulated annealing (SA), Genetic algorithm (GA) etc. however, these techniques have some limitations hence Particle Swarm optimization (PSO) technique was introduced to solve the portfolio section problem. Fuzzy approach lacks leaning ability; artificial neural network technique has over fitting problem and is easily trapped into local minimal. [11]

3 METHODOLOGY

Particle swarm optimization technique was used in this study. Considering the mean-variance model another sets of constraints were introduced. These are cardinality and bounding constraints. Assume that a set portfolio containing different assets, with a set budget, and various percentages of expected rate of returns, there is the maximum and minimum amount of capital that can be invested in each asset. This is known as the upper and lower bound. The returns and risk of each of the portfolios were computed. From the results obtained, statistical tool was engaged to evaluate the output, the minimum and maximum results were analyzed.

Python version 2.7.14 was used for program implementation because most of the libraries imported is supported by this version of python and the newer versions have a slightly different syntax. Python is a popularly used high level programming language which enables programmers to express their codes in fewer lines by eliminating the use of curly brackets and a syntax that allows programmers write codes in fewer lines.

The web page interface: The web interface has a GUI that enables users enter values such as (assets, expected rate of return, lower bound, upper bound etc.) and by clicking on the **submit** button on the interface (the route.py is triggered) these values are passed into the function (as arguments i.e assets= request.args ['num_assets'] and portfolio = request.args['budget']) to be solved.

The weights, returns and risk is then passed back to the index.html for rendering (displaying the result on the GUI).

Libraries: Most of the imported libraries are used to handle the complex mathematical calculations associated with the Markowitz formula based on cardinality and bounding constraint.

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3.1 Standard Portfolio Selection Model (Mean Markowitz Model)

Markowitz mean-variance model can be express thus:

$$\min \sigma 2p = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \qquad (1)$$

subject to $r_p = \sum_{i=1}^{N} w_i r_i \geq R$ (2)

$$\sum_{i=1}^{N} x_i = 1, \qquad (3)$$

$$w_i \geq 0, \quad \forall i \in \{1,2,....,N\} \qquad (4)$$

Where

N it stands for the number of assets in the portfolio

r_p this denotes the expected rate of returns from the assets

r_i this stands for the expected rate of return of asset i in the portfolio

σ_{ij} This stands for the covariance of returns of asset i *and* j

R This represents the investor's expected rate of return

σ^2_p stands for the return variance of the portfolio

w_i stands for the decision variable that represents the weight of budget to be invested in asset i

i represents the index of securities.

3.2 Modified Constrained Portfolio Selection Model (Cardinality And Bounding)

To improve on the mean-variance model, cardinality and bounding constraints was added to the mean-variance model that was proposed by Markowitz. Hence the question below:

$$\min \sigma 2p = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \qquad (5)$$

$$w_i = \frac{w_i c_i z_i}{\sum_{j=1}^{N} w_i c_i z_i}, \quad i = i.....N \qquad (6)$$

And

$$\sum_{j=1}^{N} z_i = M \leq N; \quad M, N \in N \qquad (7)$$

$$\sum_{i=1}^{N} x_i c_i z_i r_i \geq BR \qquad (8)$$

$$\sum x_i c_i z_i r_i \leq B \qquad (9)$$

$$0 \leq Blower \leq x_i c_i \leq Bupper \leq B, i = 1,.....N$$
(10)

$$\sum_{j=1}^{N} w_i = 1 \qquad (11)$$

$$w_i \geq 0, \quad \forall i \in \{1,2,....,N\} \qquad (12)$$

$$\sigma 2p = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \qquad (13)$$

where

c_i stands for the minimum transaction lots for asset I in the portfolio.

z_i ranges between {0,1}and it represents a binary variable, if Z is 1, then it means that asset i is in the portfolio but if Z is 0, then it not available in the portfolio.

M stands for the quantity of assets to be selected from the total number of assets available in the portfolio.

x_i c_i represents the number of units of asset i in the selected portfolio.

B stands for the total budget from the investor.

R represents the investor's expected rate of return.

B_{loweri} stands for the least amount of money an investor can budget in asset i.

B_{upperi} represents the height amount of money an investor might want to budget in asset i.

x_i represents the quantity of c_i's that is purchased.

Equation (6) represents cardinality constraint, while Equation (9) stands for the budget constraint meaning that the upper bound must not be above the total budget and Equation (10) represents the bounding constraint.

4 RESULTS AND DISCUSSION

A set of experimental results obtained from literature [12] containing 9, 30 and 50 stocks using genetic algorithm were used to compare the results from the modified constrained portfolio model using particle swarm optimization. Also, the upper and lower bound for the budget were included. The number of assets in the portfolio are 31 and 85. An assumed budget of 5000 was set and the expected rate of return was set at 5 and 10 percent respectively.

Table 1: Results of applying Genetic Algorithm to 9 stocks data set in Soleimani et al., (2009)

Size of portfolio	Expected rate of returns	5	6	7	8	9	10	11
2	Best (lowest) variance	2.89E_08	5.57E_09	7.55E_09	1.11E_07	4.59E_09	2.05E_09	1.67E_08
	Mean variance	9.343E_05	3.9055E_05	7.7596E_05	0.00023364	0.00103681	0.00033924	0.00076369
	Standard deviation of variances	0.0001992	9.3783E_05	0.00012749	0.0008117	0.00284854	0.00103222	0.00255865
3	Best (lowest) variance	0.0050611	0.00505221	0.00514064	0.00505827	0.00507855	0.00507083	0.00505425
	Mean variance	0.0067909	0.00667328	0.00717176	0.00735858	0.00636324	0.00632523	0.00648158
	Standard deviation of variances	0.0016848	0.00169722	0.00178479	0.00177102	0.00162639	0.00176300	0.00191291
4	Best (lowest) variance	3.135E_05	9.0203E_05	7.3351 E_05	2.1175E_05	9.42964E_05	6.42387E_05	0.00048999
	Mean variance	0.0007768	0.00082257	0.00099305	0.00140161	0.00088428	0.00096978	0.00287112
	Standard deviation of variances	0.0007250	0.00063435	0.00070735	0.00158779	0.00071776	0.00086213	0.00218174

Table 2: Results of applying PSO to 9 stocks data set obtained adding cardinality and bounding constraints.

Size of portfolio	Expected rate of returns	5	6	7	8	9	10	11
2	Best (lowest) variance	0.718447923	0.0333566	0.0333563	0.033356444	0.033356787	0.0333563	0.03335699
	Mean variance	0.033443762	0.0334564	0.0334553	0.033448806	0.033442825	0.0334458	0.03344861
	Standard deviation of variances	0.000133786	0.0001602	0.00015486	0.000149127	0.000136146	0.00013147	0.00014238
3	Best (lowest) variance	0.01338031	0.0133803	0.01338031	0.013389766	0.01338031	0.01338031	0.01338031
	Mean variance	0.015939147	0.0159391	0.01573788	0.016299228	0.016159836	0.01620666	0.0165096
	Standard deviation of variances	0.003727171	0.0037272	0.00308365	0.004718026	0.00433212	0.00405807	0.0049953
4	Best (lowest) variance	0.011083124	0.0110873	0.50489843	0.011087265	0.011087265	0.01108726	0.01108312
	Mean variance	0.013680518	0.0137993	0.01402056	0.014074622	0.014147731	0.01429344	0.01404251
	Standard deviation of variances	0.003598778	0.0038331	0.0045259	0.004571174	0.004675406	0.00484591	0.00501054

In table 2, the PSO algorithm were compared using three data sets 9, 30 and 50 stocks that was originally used in [12]. This is done in order to make the problem more accurate and complex. Also, upper and lower bound were included. The results obtained in tables 1 and 2 confirm that when the size of the asset is increased, the best risk value obtained using PSO are better. For each of the parameters considered, particle swarm optimization gives a better result than the results obtained from the genetic algorithm by [12]. This will off course inform an investor's decision-making ability. Since particle swarm optimization gives a better solution to the problem been solved.

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Table 3: Comparison of Constrained PSO and GA for data set of 9 stocks.

Size of portfolio	Expected rate of returns	5	6	7	8	9	10	11
2	Best (lowest) variance	-2.49E+09	-6.E+08	-4.42E+08	-3.01E+07	-7.27E+08	-1.63E+09	-2.00E+08
	Mean variance	-3.57E+04	-8.56E+04	-4.30E+4	-1.42E+04	-3.13E+03	-9.76E+03	-4.28E+03
	Standard deviation of variances	32.8384	-7.08E+01	-2.15E+01	8.16E+01	9.52E+01	8.73E+01	9.44E+01
3	Best (lowest) variance	-164.37553	-164.841	-160.285	-164.71	-163.467	-163.868	-164.734
	Mean variance	-134.713	-138.85	-119.442	-121.5	-153.956	-156.222	-154.716
	Standard deviation of variances	-121.2233	-119.6062	-72.7738	-166.402	-166.364	-130.18	-161.136
4	Best (lowest) variance	-3.53E+04	-1.22E+04	9.31E+01	-5.23E+04	-1.17E+04	-1.72E+04	-2.16E+03
	Mean variance	-1.66E+03	-1.58E+03	-1.31E+03	-9.04E+02	-1.50E+03	-1.37E+03	-3.89E+02
	Standard deviation of variances	-3.96E+02	-5.04E+02	-5.40E+02	-1.88E+02	-5.51E+02	-4.62E+02	-1.30E+02

The comparisons of the results are performed based on four criteria. These criteria are: best (lowest) variance (risk) among the risks obtained from the algorithm runs, showing the best solution found, mean variance, the average of the value of the objective function found by the algorithm, standard deviation of variances, showing how solutions found by the algorithm are close to each other.

Table 4: Shows the Returns and risk when the size of the portfolio is 31 and expected rate of return is 5%.

S/N	Returns	Risk
1	0.391789384	0.042736372
2	0.391789384	0.042736372
3	0.537598601	0.052669575
4	0.504007668	0.050593809
5	0.339446155	0.03822803
6	0.442786693	0.046507105
7	0.310166116	0.03522052
8	0.297297884	0.033749235
9	0.285089391	0.032243203
10	0.271855609	0.030495983
11	0.33978219	0.038260238
12	0.324567058	0.036753378
13	0.242677216	0.0260707
14	0.23542542	0.024769068
15	0.228337454	0.023382794
16	0.221875625	0.022005896
17	0.216193826	0.020686661
18	0.210973663	0.019358696
19	0.242840703	0.02609883
20	0.235571751	0.024796479
21	0.19842522	0.015497482
22	0.222004785	0.022034597
23	0.192031215	0.013017167
24	0.18952613	0.011880632
25	0.206675365	0.01816216
26	0.185652871	0.009841476
27	0.184271305	0.008984901
28	0.183166461	0.008221478
29	0.192084382	0.013040062
30	0.181580125	0.006934667
Min	**0.181019**	**0.006394205**
Max	**0.537599**	**0.052669575**
STDV	**0.094524**	**0.012983705**
Average	**0.266491**	**0.025287847**

Table 4 shows the results obtained when the of returns was set to 5 percent on 31 stocks the budget was set to 5000, the upper bound and lower bound was set to 1000 and 5000 respectively.

From the result obtained in the above table 3, it depicts the solutions are not very satisfactory although investors will always want to invest in assets with minimum risk and maximum returns. Positive values depict the improvement obtained in percentage while using PSO compared to genetic algorithm (GA).

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Table 5: Shows the Returns and risk when the size of the portfolio is 85 and expected rate of return is 5%.

S/N	Returns	Risk
1	0.523527858	0.067161759
2	0.355771645	0.056486328
3	0.338327616	0.055047448
4	0.439840459	0.062414242
5	0.417733726	0.060994122
6	0.28676608	0.05026034
7	0.271629004	0.048668946
8	0.257577345	0.04709747
9	0.338678448	0.05507732
10	0.23007236	0.04373096
11	0.217579981	0.042061725
12	0.287079777	0.050292316
13	0.195405478	0.03883027
14	0.186332635	0.03737108
15	0.243972892	0.04548097
16	0.170730778	0.034580887
17	0.217833862	0.042096647
18	0.158081972	0.031970146
19	0.195604128	0.038861159
20	0.186507349	0.037400144
21	0.178594335	0.036037351
22	0.138115765	0.026722951
23	0.16402948	0.033247932
24	0.129808164	0.023926314
25	0.126403769	0.022623155
26	0.14768101	0.029450571
27	0.121000322	0.020253319
28	0.118085628	0.018780786
29	0.115545277	0.017375288
30	0.113346196	0.016036304
Min	**0.113346**	**0.016036**
Max	**0.523528**	**0.067162**
Standard deviation	**0.103796**	**0.01383**
Average	**0.229055**	**0.039678**

Table 5 presents the results obtained when the of returns was set to 5 percent on 85 stocks the budget was set to 5000, the upper bound and lower bound was set to 1000 and 500 respectively.

Table 6: Shows the Returns and risk when the size of the portfolio is 31 and expected rate of return is 10%.

S/N	Returns	Risk
1	0.575920446	0.054928729
2	0.371868943	0.041127655
3	0.355206403	0.039689865
4	0.47253525	0.048543393
5	0.324274975	0.036723365
6	0.310166116	0.03522052
7	0.392237921	0.042771497
8	0.285089391	0.032243203
9	0.355524882	0.03971834
10	0.33978219	0.038260238
11	0.250849287	0.027421534
12	0.242677216	0.0260707
13	0.297545059	0.033778537
14	0.228337454	0.023382794
15	0.27211552	0.030531455
16	0.260576789	0.028902566
17	0.251027767	0.027449835
18	0.242840703	0.02609883
19	0.235571751	0.024796479
20	0.19842522	0.015497482
21	0.222004785	0.022034597
22	0.216299645	0.020712368
23	0.18952613	0.011880632
24	0.187377666	0.010802564
25	0.202883193	0.016998503

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26	0.184271305	0.008984901
27	0.194954621	0.014212155
28	0.192084382	0.013040062
29	0.189575426	0.011904132
30	0.187417351	0.010823548
Min	**0.184271**	**0.0089849**
Max	**0.57592**	**0.0549287**
Standard deviation	**0.091295**	**0.0120734**
Average	**0.274299**	**0.0271517**

Table 6 shows the results obtained when the of expected rate of returns was set to 10 percent on 31 stocks the budget was set to 5000, the upper bound and lower bound was set to 1000 and 500 respectively.

Table 7: Shows the Return and risk when the size of the portfolio is 85 and expected rate of return is 10%

S/N	Returns	Risk
1	0.523527858	0.067161759
2	0.49294367	0.065511796
3	0.338327616	0.055047448
4	0.439840459	0.062414242
5	0.417733726	0.060994122
6	0.28676608	0.05026034
7	0.271629004	0.048668946
8	0.05651609	0.356144643
9	0.243685454	0.045445794
10	0.23007236	0.04373096
11	0.217579981	0.042061725
12	0.287079777	0.050292316
13	0.271926931	0.048701219
14	0.257862322	0.047130319
15	0.243972892	0.04548097
16	0.170730778	0.034580887
17	0.163900414	0.033221255
18	0.158081972	0.031970146
19	0.152660789	0.030707264
20	0.147578805	0.029423697
21	0.178594335	0.036037351
22	0.138115765	0.026722951
23	0.16402948	0.033247932
25	0.129808164	0.023926314
26	0.15276997	0.030733723
27	0.14768101	0.029450571
28	0.121000322	0.020253319
29	0.138206599	0.026751088
30	0.115545277	0.017375288
Min	**0.113346**	**0.016036**
Max	**0.523528**	**0.067162**
Standard deviation	**0.113077**	**0.014144**
Average	**0.235705**	**0.040329**

Table 7 shows the results obtained when the of returns was
set to 10 percent on 85 stocks the budget was set to 5000,
the upper bound and lower bound was set to 1000 and 500 respectively

Table 8: Summary of results obtained when the size of the portfolio is 31 and 85

Portfolio	Expected rate	5 Returns	Risk	10 Returns	Risk
31	Min	0.181019	0.0063942	0.184271	0.0089849
	max	0.53759	0.0526695	0.57592	0.0549287
	Standard deviation	0.094524	0.0129837	0.091295	0.0120734
	Average	0.266491	0.0252878	0.274299	0.0271517
	Min	0.113346	0.0016036	0.113346	0.016036
85	Max	0.523528	0.067162	0.523528	0.067162
	Standard deviation	0.103796	0.01383	0.113077	0.014144
	Average	0.229055	0.039678	0.2335705	0.040329

The above table shows the summary of the results obtained when the size of the portfolio is 31 and 85 at 5 and 10 percent rate of returns. From the results obtained, when the value of the risk tending towards zero (0) the investors sensitivity is increased as this will motivate the investor to invest in the portfolio where the risk is minimum. Then on the other way around, when the returns is tending to one (1) or unity, the investor is attracted to invest in such portfolio since the aim of an investor is to make maximum returns.

From the above analysis, it therefore means that when the expected rate of returns is high, the risk on investment is increased while the return remains constant. It also revealed that this model preforms better when the size of the portfolio is small as seen in table 4.8 when the size of the stock is 31, the minimum returns is high, and the risk is low when compared with 85 stocks same for all the other parameters considered in this study.

5. CONCLUSION

Portfolio selection is a trivial issue in today's banking system. Portfolio selection models has been

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helpful in many areas like credit portfolio management, decision making, financial institutions management etc. This technique has been helpful in taking critical decisions, enhancing profitability, risk management etc. Although several algorithms have been used for portfolio selection problem, but these algorithms are not enough to solve the portfolio selection problem and there are some constraints associated with portfolio optimization.

In this study cardinality and bounding constraints were used to build portfolio selection model using a meta-heuristic technique of particle swarm optimization. The results of the study were compared with that of the genetic algorithms

obtained with the model developed shows that particle swarm optimization approach gives a better result than genetic algorithm in solving portfolio selection problem.

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