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Modelling and Control of Flow Induced Vibration of Top-Tensioned Marine Risers using Analytical Methods

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Abstract

For a riser array in deep waters, interference between individual risers in strong ocean current is of operational concern and thereby a key design issue. The lateral deflection is likely to be large, and the risers may experience collision with fatigue or coating damage as a consequence. In this paper, active control of flexible marine riser angle and the reduction of flow induced (forced) vibration under a time varying distributed load were considered using boundary control approach. A torque actuator was introduced in the upper riser package and a boundary control was designed to generate the required signal for riser angle control and vibration reduction with guaranteed closed-loop stability. The design is based on the partial differential equations of the system, which are developed using energy principle. Analytical method of solution was deployed with the aid of a program, developed within the framework of MATLAB, to predict the riser's behaviour by top tensioning. A sensitivity analysis for different values of the control variables was carried out. The results of this work showed that active control of flexible marine riser by top-tensioning reduced flow induced vibration.

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1. Introduction

The riser plays a very important role in oil drilling and production offshore as shown in Fig 1 and Fig. 2. A marine riser is the connection between a platform on the water surface and the well head on the seafloor. Tension is applied at the top of the riser which allows it to resist lateral loads. Its effects on natural frequencies; mode shapes and forced vibration have been studied in Often. [1]

Also in conventional approaches for control design, an approximate finite dimensional model is used. The approximate can be obtained via spatial discretization to obtain a finite number of modes or by modal analysis and truncating the infinite number of modes to a finite number by neglecting the higher frequency modes. Based on a truncated model obtained from either the finite element method or assumed modes method, various control

approaches have been applied to improve the performance of flexible systems [2]. However, spill over effects from the control to the residual modes, which results in instability have been observed in [3] when the control

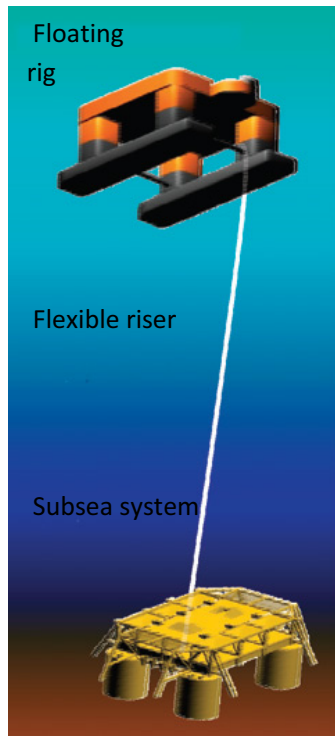


Figure 1 The Marine riser.

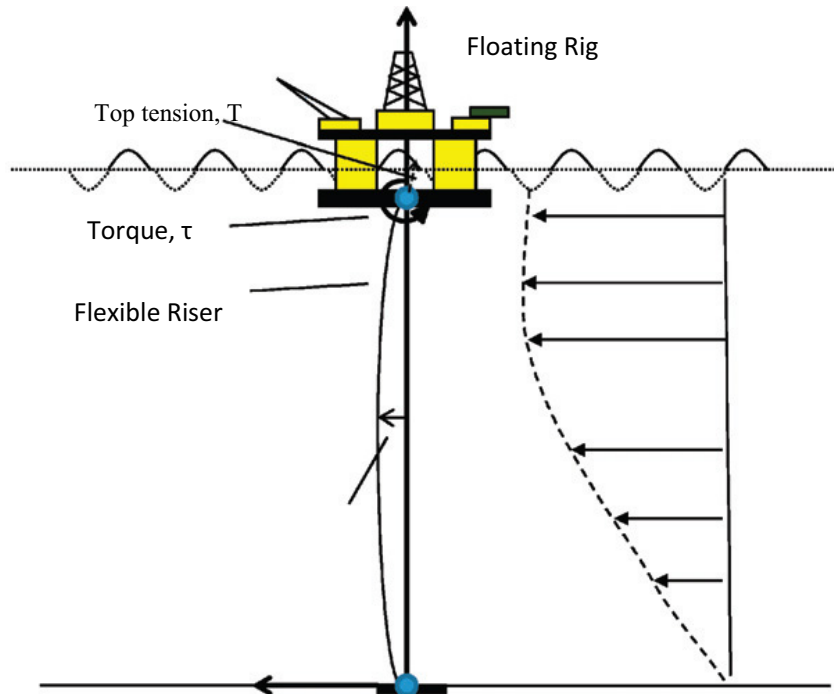


Figure 2. Schematics

of the truncated system is restricted to a few critical modes. To avoid the problems associated with the truncated-model-based design, control methodologies such as boundary control can be used. Boundary control has been applied to beams in [4,5] where boundary feedback was used to stabilize the wave equations and design active constrained layer damping. Active boundary control of an Euler-Bernoulli beam which enables the generation of a desired boundary condition at any designators position of beam structure has been investigated in different research works.[6] However in this study, the objective is to reduce the deflection at the top joint and simultaneously reduce the vibrations of the riser.

A riser system is essentially conductor pipes connecting floaters on the surface and the wellheads at the seabed.[7,8] There are essentially two kinds of risers, namely rigid riser and flexible riser. A hybrid riser is the combination of these two. There are a variety of possible configurations for marine risers, such as free hanging catenary riser, top tensioned production riser, lazy S riser, steep wave riser and pliant wave riser (API RP 2RD,). For deep water production systems, riser solutions are traditionally divided into two main groups; (1) subsea wellheads (wet trees) with flexible risers up to a floater like a semi-submersible or a production ship (floating production storage offloading - FPSO), and (2) tensioned risers with wellhead on a compliant platform, like tension leg platform (TLP), spar1 or deep draft floater (DDF). (API RP 2RD,). These are the dry tree solutions for floating production systems. Dry tree systems are often the preferred solution for production as they provide easy access to the well for maintenance, intervention and work over. Alternative platform solutions, while still using dry tree systems, are proposed for a semisubmersible with heave compensation system, and for deep draft caisson vessel with buoyancy cans. Pollack suggests a weight based tension leg deck (TLD) to which the risers are locked for a dry tree solution on FPSOs. This solution allows long stroke for spread moored ship-shaped vessels.[9] A two tier well

riser tension system, which is a mechanical alternative to hydraulic cylinders working in series, have already been presented.

This system separates the low frequency stroke motion from the wave induced stroke motion.

Top tensioned risers operated from spars and TLPs are arranged in clusters of (near) vertical riser arrays. The number of individual risers in an array may be 20 or more, which may consist of different risers applied for production, drilling, work over, export, etc. The problem of riser collision can be restricted to tensioned risers operated from a TLP or a spar located in deep waters. This is due to the small space available for the high number of risers, in addition to the increased deflection for long risers.[10,11,12]

A TLP is chosen as the platform concept for this work, as shown in figures 1 and 2. The main reason is that tensioned risers can be applied at a TLP with a relatively small requirement for stroke capacity even at large TLP offsets. This is a consequence of the geometric restrictions for heave motions caused by the axially rigid tendons.[13,14,15]. For other floaters like spars, FPSOs and semi-submersibles, the demand for stroke capacity will be much higher, which means that other riser solutions like steel catenary risers (SCR) or flexible risers are preferred. Hence, the control system presented in this dissertation is most likely to be applied on a TLP..

2. Governing Equation

The equation of motion is in linearized form as

$$EI \frac{d^4 y}{dx^4} - T \frac{d^2 y}{dx^2} + m_z \frac{d^2 y}{dt^2} + C \frac{dy}{dt} = f(x, t) = \frac{1}{4} \rho C_D U_m^2 (1 + \cos 2\omega t) \tag{1}$$

The governing equation (1) for the flexible marine riser, is a fourth-order PDE with axial tension, structural damping and external disturbances terms, remains in the same form as considered in [16]

The initial and boundary conditions are respectively:

$$y(x, 0) = y^0, y^1(x, 0) = 0 \tag{2}$$

$$y(0, t) = EI \frac{d^2 y(0,t)}{dx^2} = 0 \tag{3}$$

$$y(L, t) = EI \frac{d^2 y(L,t)}{dx^2} = 0 \tag{4}$$

3. Problem solution

Applying Laplace transform to the damped free vibration equation (1) we have

$$EI \frac{d^4 \check{y}}{dx^4} - T \frac{d^2 \check{y}}{dx^2} + M_z [s^2 \check{y}(x, s) - S y(x, 0) - y'(x, 0)] + C [S \check{y}(x, s) - y(x, 0)] = 0 \tag{5}$$

On putting the initial conditions equation (5) reduces to

$$EI \frac{d^4 \check{y}}{dx^4} - T \frac{d^2 \check{y}}{dx^2} + (M_z S^2 + CS) \check{y}(x, s) - (M_z S + C) - y_0 = 0 \tag{6}$$

Applying Fourier Finite Sine transform to equation (6)

$$\check{\check{y}}(n, s) = \frac{[1+(-1)^{n+1}]y_0 L (M_z S+C)}{n\pi \left[EI \left(\frac{n^2 \pi^2}{L^2} \right)^2 + T \frac{n^2 \pi^2}{L^2} + M_z S^2 + CS \right]} \tag{7}$$

Which can be written as

$$\check{\check{y}}(n, s) = \frac{1+(-1)^{n+1}y_0 L}{n\pi} \left[\frac{S + \frac{C}{m_z}}{\left(S + \frac{C}{2m_z} \right)^2 + \frac{1}{M_z} \left[\left(\frac{n\pi}{L} \right)^2 \left[EI \left(\frac{n\pi}{L} \right)^2 + T \right] \right]} \right] \tag{8}$$

For the case of undamped free vibration, C = 0, which makes equation (8) to reduce to

$$\tilde{y}(n, s) = \frac{1+(-1)^{n+1}y_0 L}{n\pi} \left[\frac{S}{S^2 + \frac{1}{m_z} \left(\frac{n\pi}{L}\right)^2 \left[EI \left(\frac{n\pi}{L}\right)^2 + T \right]} \right] \quad (9)$$

Therefore,

$$y(n, s) = \frac{1+(-1)^{n+1}y_0 L}{n\pi} \left[\frac{S}{\left[S - \frac{i}{m_z} \left(\frac{n\pi}{L}\right)^2 \left[EI \left(\frac{n\pi}{L}\right)^2 + T \right] \right] \left[S + \frac{i}{m_z} \left(\frac{n\pi}{L}\right)^2 \left[EI \left(\frac{n\pi}{L}\right)^2 + T \right] \right]} \right] = \frac{1+(-1)^{n+1}y_0 L}{n\pi} \left[\frac{S}{(S-i\omega)(S+i\omega)} \right] \quad (10)$$

Where

$$\omega_n^2 = \frac{i}{m_z} \left(\frac{n\pi}{L}\right)^2 \left[\left(\frac{n\pi}{L}\right)^2 + T \right] \quad (11)$$

$$\text{Which implies, } \omega_n = \frac{n\pi}{L} \sqrt{\frac{1}{m_z} \left[\left(\frac{n\pi}{L}\right)^2 + T \right]} \quad (12)$$

Equation (12) represents the natural frequency of the undammed free vibration.

For the damped free vibration, the poles of equation (1) can be written as follows

$$\begin{aligned} \left(S + \frac{c}{2M_z} \right)^2 + \frac{1}{M_z} \left(\frac{n\pi}{L}\right)^2 \left[EI \left(\frac{n\pi}{L}\right)^2 + T \right] &= \left(S + \frac{c}{2M_z} \right)^2 + \omega_{nc}^2 \\ &= \left[S + \frac{c}{2M_z} - \frac{i}{M_z} \left(\frac{n\pi}{L}\right)^2 \left[EI \left(\frac{n\pi}{L}\right)^2 + T \right] \right] \left[S + \frac{c}{2M_z} + \frac{i}{M_z} \left(\frac{n\pi}{L}\right)^2 \left[EI \left(\frac{n\pi}{L}\right)^2 + T \right] \right] \end{aligned} \quad (13)$$

$$= (S - i\omega)(S + i\omega) \quad (14)$$

$$\text{This implies, } \omega^2 = -\frac{c}{2M_z} + \frac{i}{M_z} \left(\frac{n\pi}{L}\right)^2 \left[EI \left(\frac{n\pi}{L}\right)^2 + T \right] \quad (15)$$

4. Results And Discussions

The riser system was modelled in this paper. The result of the simulation of closed-loop system (equation 6), that shows the input torque at the upper riser end modelled as a boundary condition in relation to the dynamics of the system is presented. This was done to investigate the performance of control law, system parameters and the control objectives proposed for Marine risers. For the simulations, 1000m water depth was used.

The riser model was verified with a variety of current profiles. The simulation of the model was done using MATLAB software and the results of the simulation are presented. These results are discussed based on the cases.

First the simulation case was run without control, see the figures, showing overlay of riser profiles without control. In this case the no application of top tension. As depth and current increase the risers are seen to slide out to the right towards the middle of the depth. The maximum deflection is seen at the 500m above seabed. Collision occurs along almost the entire riser.

In addition, showing the sea current effect on the risers with time and the maximum lateral displacement of risers with time. More so varying the masses, since the response of dynamical system is dependent on mass factor, hence the three masses of 15 kg, 45 kg and 75 kg and their responses.

Riser displacement at $x = 200$ m (at platform), Overlay of riser profiles without control (Red solid) and, with control. It can be observed that variations on the riser top tension modify riser displacement response. It happens because variations of the riser top tension modify the riser geometric stiffness. In the figures below, it can be noted that if riser top tension increases the riser stiffness increases, and the overall riser displacement response decreases. Moreover, from Fig.7 and Fig. 8, if riser top tension increases, riser displacement along its length decreases. Riser displacement at $x = 400$ m (at platform), Overlay of riser profiles without control (Red solid) and, with control. It can be observed that variations on the riser top tension modify riser displacement response. It happens because variations of the riser top tension modify the riser geometric stiffness. In Fig.7, it can be noted that if riser

top tension increases the riser stiffness increases, and the overall riser displacement response decreases. Riser displacement at $x = 500$ m (at platform), Overlay of riser profiles without control (Red solid) and, with control . It can be observed that variations on the riser top tension modify riser displacement response. It happens

because variations of the riser top tension modify the riser geometric stiffness. In Fig.8, it can be noted that if riser top tension increases the riser stiffness increases, and the overall riser displacement response decreases.

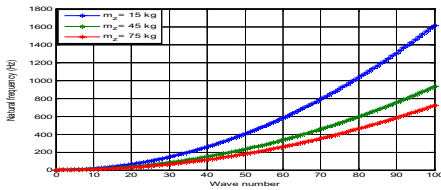


Fig.3. Wave number

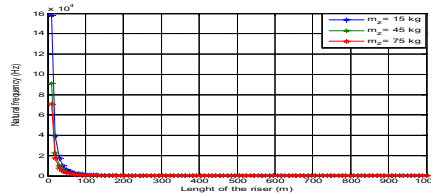


Fig.4. The natural frequency

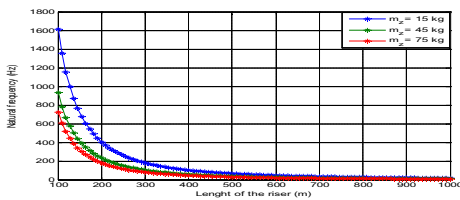


Fig.5. The natural frequency with increasing length of the riser

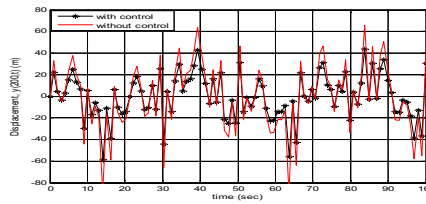


Fig.6. Riser displacement at $x = 200$ m

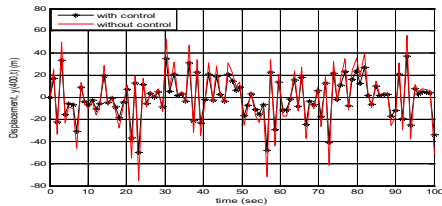


Fig.7. Riser displacement at $x = 400$ m

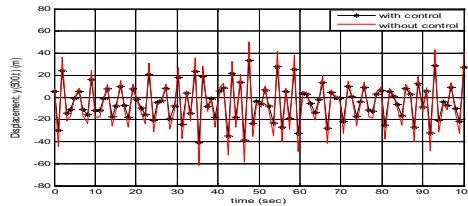


Fig.8. Riser displacement at $x = 500$ m

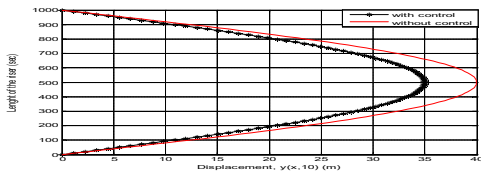


Fig.9. The controlled and uncontrolled upper and lower riser angles

Considering Simulation with control, the risers are exposed to the same environmental set-up, but now control of top tension is introduced. In this case the active control of top tension is in place by tensioning. As depth and current increase the risers are seen to slide out to the right towards the middle of the depth. The controlled and uncontrolled upper and lower riser angles are shown in Figure 9. It is observed that there are significant improvements in the top riser angle bringing the magnitude near zero when the control is applied. There are also some peak angles reductions in the bottom angle though the actuator is not located at that position. Also shown is sea current effect on the risers

with time, maximum lateral displacement of risers with time, Natural frequencies of risers at different top-tension with respect to Wave number and Hydrodynamic force acting on risers with respect to time.

5. Conclusion

This paper was focused on modelling, simulation and control of flow-induced top tensioned marine risers. Specifically, we considered how to reduce the distance (lateral displacement) of risers. This translates to distance between risers in an array, exposed to platform motions. Current and hydrodynamic interaction forces by controlling the top tension.

In this paper, the model of a flexible marine riser with a torque actuator at the upper riser package was derived. Boundary control was introduced to reduce the upper riser angle magnitude and the transverse vibration of a riser subjected to a distributed load. Closed-loop stability was proven directly from the Partial differential equation (1) of the system modelled by analytical method and the problem of traditional truncated-model-based design was avoided. The disturbance is persistent as in the case of the marine environment, the magnitude of deflection was shown to be reduced under the control action. The riser was also shown to be exponentially stabilized in the absence of external disturbance. From the simulations, we observe that there is significant improvement in the upper riser angle magnitude and the vibration reduction of the riser has been achieved.

In addition from this study is the prevention of risers' collision by use of top tension control provided the maximum displacement (spacing) was factored into the design of riser.

This included the current velocity profiles, hydrodynamic interaction between risers, the riser model and the actuator (tensioner system). Finally, comparison with the numerical simulations from Active control of Marine riser, verified the solution achieved in this work.

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