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The U-Quadratic Distribution as a Proxy for a Transformed Triangular Distribution (TTD)

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Abstract: In this study, the basic moments of the triangular distribution are studied, and the moments of the resulting distribution referred to as a Transformed Triangular Distribution (TTD) are compared with those moments of the U-quadratic distribution based on some suppositions. The results show that the U-quadratic distribution is a proxy for the TTD.

Key words: Moments, probability distribution, triangular distribution, TTD, u-quadratic distribution

INTRODUCTION

The triangular distribution is a continuous distribution known for some special properties such asfixed minimum,most likely value to occur (mode) and fixed maximum. The extreme values: fixed minimum (resp. fixed maximum) are referred to as lower limit (resp. upper limit) (Jance and Thomopoulos, 2012). For a symmetrical nature of the distribution; the mode, mean, and median are the same (Edeki and Adeosun, 2014, Paul, 1970).

In terms of application, Johnson (1997, 2002) noted that the triangular distribution is a proxy for the beta distribution in assessing risk and uncertainty, for instance, project evaluation and review. Garg *et al*., (2009) considered the sum of two triangular random variables. Notable work on triangular distribution and related areas of applied probability include those of (Back *et al*., 2000; Johnson and Kotz, 1999).

In applied probability theory, the U-quadratic distribution is a continuous probability distribution defined by a special quadratic function possesse slower limit (resp. upper limit) as. The importance of the U-quadratic distribution is easily traceable to the modelling of symmetric bimodal processes.

It is our interest in this research to present the transformed triangular distribution as new probability distribution resulting from the extreme values as average of the triangular distribution (Okagbue *et al*., 2014).

MATERIALS AND METHODS

The basic moments of the distributions: In this study, we present the basic moments and properties of the concerned distributions.

The Triangular Distribution (TD): Suppose X is a random variable with parameters a, b and c such that:

$$a : a \in (-\infty, \infty)$$
$$b : a \le b \le c$$
$$c : a < c$$

and a support, $a \le x \le b$ then, X is said to follow a triangular distribution XT(a, b, c) with a probability density function (pdf) f(x) of X given as:

$$f(x) = \begin{cases} 0 \quad , & \text{for} \quad x < a \\ \dfrac{2(x-a)}{(c-a)(b-a)}, & \text{for} \quad a \le x \le b \\ \dfrac{2(c-x)}{(c-a)(c-b)}, & \text{for} \quad a \le x \le c \\ 0 \quad , & \text{for} \quad x > c \end{cases} \tag{1}$$

The U-quadratic distribution: Let Y be a continuous random variable defined on the interval $I = [a_*, b_*]$ with a_* (resp. B_*) as lower limit (resp. upper limit), then Y follows aU-Quadraticuniform distribution with pdf g defined and denoted as follows:

$$g\left(y \middle| a_*, b_*, \eta, \zeta\right) = \begin{cases} \eta\left(y-\zeta\right)^2, \; a_* \le x \le b_* \\ 0, \text{otherwise} \end{cases} \tag{2}$$

Where:

$$\zeta = \frac{b_* + a_*}{2} \text{ and } \eta = \frac{12}{\left(b_* - a_*\right)^3} \tag{3}$$

The transformation of the Triangular Distribution (TTD): Replacing the most likely value, with the average of the maximum and minimum of the Triangular distribution toward modifying the Triangular distribution, gives:

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Table 1: Properties and moments of U-quadratic and TTD

Distribution	U-Quadratic	TTD
Parameters	$a_*: a_* \in (-\infty,\infty)$, $b_*: b_* \in (a,\infty)$, $\eta: \eta \in (a,\infty)$, $\zeta: \zeta \in (-\infty,\infty)$	$a: a \in (-\infty,\infty)$, $b: b \in [a,c]$, $c: a < c$
Support	$y \in [a_*, b_*]$	$x \in [a,b]$
pdf	$g(y \mid a_*, b_*, \eta, \zeta),\ g(y) = \begin{cases} \eta(y-\zeta)^2, & a_* \le x \le b_* \\ 0, & \text{otherwise} \end{cases}$	$f^*(x) = \begin{cases} \frac{4(x-a)}{(c-a)^2}, a \le x \le \frac{a+c}{2}, \frac{4(c-x)}{(c-a)^2}, \\ \frac{a+c}{2} \le x \le c,\ 0, \text{otherwise} \end{cases}$
Mean	$X^U_{mean} = \frac{b_* + a_*}{2}$	$X^*_{mean} = \frac{a+c}{2}$
Median	$X^U_{median} = \frac{b_* + a_*}{2}$	$X^*_{median} = \frac{a+c}{2}$
Mode	$X^U_{mode} = b_* = \frac{a_* + b_*}{2}$	$X^*_{mode} = b = \frac{a+c}{2}$
Variance	$Var(UQ) = \frac{3(b_* - a_*)^2}{20}$	$Var(TTD) = \frac{(c-a)^2}{24}$
Skewness	$sk\_UQ = 0$	$sk\_TTD = 0$

$$2b = c + a \quad (4)$$

This leads to new distribution the Transformed Triangular Distribution (TTD).

The resulting distribution TTD and its properties: For a random variable X_* associated with the TTD, f^* (x)the corresponding pdf is defined as:

$$f^*(x) = \begin{cases} \frac{4(x-a)}{(c-a)^2}, & a \le x \le \frac{a+c}{2} \\ \frac{4(c-x)}{(c-a)^2}, & \frac{a+c}{2} \le x \le c \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

A note on the distributions and definitions of basic moments: The TTD pdf has been proven for validity, the moments and properties of same are presented by Okagbue *et al*. (2014). We claim here, that the support and parameters of the TD and TTD coincide. In what follows, the basic moments of the distributions are defined and denoted as, X^u_{mean}, (resp. X^*_{mean}), X^u_{mean} (resp. X^*_{mean}) X^u_{mode} (resp. X^*_{mode}) Var (U_Q) (resp. Var TTD), sk_UQ (resp. sk-TTD) for mean, median, mode, variance, and skewness for u-quadratic distribution (resp. TTD). Generally, such is denoted by M^0_{UQ} and M^0_{TTD}, respectively. In Table 1, we show the different distributions (U_quadratic and TTD) with respect to their parameters, supports and basic moments.

RESULTS AND DISCUSSION

Moments and support parameters: In generally, let the basic moments of the U-quadratic distribution and TTD be denoted by M^0_{UQ} and M^0_{TTD} respectively. Suppose c = b_* and k ∈ [0, 5/18], then the following can easily be shown: X^u_{mean}, X^*_{mean} X^u_{mediam} X^*_{medium}, X^u_{mode} X^*_{mode}, sk_ UQ sk _TTD, and kvar (U_Q) Var and.

CONCLUSION

In this, study the basic moments of the distributions: triangular, transformed triangular, and U-quadratic were studied, and comparison between the moments of TTD and of those of the U-quadratic distribution was made. Based on some suppositions,the results showed that the U-quadratic distribution is a proxy for the TTD.

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